

HW 2

1. 46 2) a) $dV_1 = \left(\frac{\partial V}{\partial T}\right)_P dT = \beta V dT$, where $\beta = (1/V)(\partial V/\partial T)_P$

2) b) $dV_2 = \left(\frac{\partial V}{\partial P}\right)_T dP = -k_T V dP$

2) c) $dV_1 + dV_2 = 0$

$$k_T V dP = \beta V dT$$

$$\left(\frac{\partial P}{\partial T}\right)_V = -\frac{\beta}{k_T} = -\frac{(\partial V/\partial T)_P}{(\partial V/\partial P)_T}$$

3) d) For ideal gas

$$\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{V} \frac{\partial}{\partial T} \left(\frac{NkT}{P}\right) = \frac{1}{V} \frac{NK}{P} = \frac{1}{T}$$

$$k_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P}\right)_T = -\frac{1}{V} \frac{\partial}{\partial P} \left(\frac{NkT}{P}\right) = -\frac{1}{V} \left(-\frac{NkT}{P^2}\right) = \frac{1}{P}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{1}{\partial T} \left(\frac{NkT}{V}\right) = \frac{NK}{V} = \frac{P}{T} = \frac{\beta}{k_T}$$

3) e) The pressure change for equal-V:

$$\text{Water: } dP = \left(\frac{\partial P}{\partial T}\right)_V dT = \frac{\beta}{k_T} dT = \frac{2.57 \times 10^{-4} \text{ K}^{-1}}{4.52 \times 10^{-10} \text{ Pa}^{-1}} \cdot 10 \text{ K} \approx 56 \text{ atm}$$

$$\text{Mercury: } dP = \frac{\beta}{k_T} dT = \frac{1.81 \times 10^{-4} \text{ K}^{-1}}{4.04 \times 10^{-11} \text{ Pa}^{-1}} \cdot 10 \text{ K} \approx 440 \text{ atm}$$

Large pressure requires expensive equipment.

I prefer to measure C_p instead of C_V .

$$2.15 \quad 50! = 3.0414 \times 10^{64}$$

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Sterling Approximation:

$$N! \approx N^N e^{-N} \sqrt{2\pi N} = 50^{50} e^{-50} \sqrt{2\pi \cdot 50} = 3.0363 \times 10^{64}.$$

0.2% difference.

$$\ln 50! = 148.4778$$

$$(2.16) \Rightarrow \ln 50! = 50 \ln 50 - 50 = 145.6012$$

2% difference.

$$q \ll N$$

$$2.17 \quad \ln \underline{N} \approx (N+q) \ln(N+q) - q \ln q - N \ln N$$

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$$\approx (N+q) \ln \left(N \left(1 + \frac{q}{N} \right) \right) - q \ln q - N \ln N$$

$$\downarrow = \ln N + \ln \left(1 + \frac{q}{N} \right) \approx \ln N + \frac{q}{N}$$

$$\approx N \ln N + q \ln N + (N+q) \left(\frac{q}{N} \right) - q \ln q - N \ln N$$

$$= q \ln \left(\frac{N}{q} \right) + q + \frac{q^2}{N} \approx q \ln \left(\frac{N}{q} \right) + q$$

$$\underline{N} = e^{q \ln(N/q)} e^q = \left(\frac{eN}{q} \right)^q$$

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For two-state system:

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$$\Omega = \binom{N}{N_\downarrow} = \frac{N!}{N_\downarrow! (N - N_\downarrow)!}$$

$$\text{Sterling equation } N! \approx \left(\frac{N}{e}\right)^N$$

$$\Omega \approx \frac{N^N}{N_\downarrow^{N_\downarrow} \cdot (N - N_\downarrow)^{N - N_\downarrow}}$$

$$\ln \Omega \approx N \ln N - N_\downarrow \ln N_\downarrow - (N - N_\downarrow) \ln (N - N_\downarrow)$$

$$\therefore \frac{N_\downarrow}{N} \ll 1$$

$$\therefore \ln(N - N_\downarrow) = \ln N + \ln\left(1 - \frac{N_\downarrow}{N}\right) \approx \ln N - \frac{N_\downarrow}{N}$$

$$\therefore \ln \Omega \approx N \ln N - N_\downarrow \ln N_\downarrow - (N - N_\downarrow)\left(\ln N - \frac{N_\downarrow}{N}\right)$$

$$= N_\downarrow \ln N - N_\downarrow \ln N_\downarrow + N_\downarrow = \frac{N_\downarrow^2}{N}$$

$$\approx N_\downarrow \ln \frac{N}{N_\downarrow} + N_\downarrow = N_\downarrow \ln \left(\frac{Ne}{N_\downarrow}\right)$$

$$\Omega \approx \left(\frac{Ne}{N_\downarrow}\right)^{N_\downarrow}$$

In the limit $N_\downarrow \rightarrow N$, magnet system is the same

as Einstein solid. It is because in this limit, the states of Einstein solid in which can hold more than one unit of energy are insignificant.

So the Einstein solid system is similar to the magnet system (have one unit energy OR have no energy).

2.22. a) The energy of solid A can vary from 0 to $2N$

$$\boxed{2N+1}$$

states.

b) From problem (2.18). the multiplicity of an Einstein Solid

for very large N and q is

$$\Omega(N, q) = \frac{\left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N}{\sqrt{2\pi q(q+N)/N}}$$

Now substitute q and N with $2N$. (Because we have a $2N$ oscillators system with total energy units $2N$)

$$\Omega_{\text{total}}(N, q) = \frac{2^{4N}}{\sqrt{8\pi N}}$$

c) Only for solid A, which has N oscillators and energy N

$$\Omega_A(N, q) = \frac{\left(\frac{N+N}{N}\right)^N \left(\frac{N+N}{N}\right)^N}{\sqrt{2\pi(N+N)/N}} = \frac{2^{2N}}{\sqrt{4\pi N}}$$

$$\Omega_{\text{most like}} = \Omega_A \times \Omega_B = \frac{2^{4N}}{4\pi N}$$

$$d) \text{ width } \approx \frac{\Omega_{\text{total}}}{\Omega_{\text{most like}}} = \sqrt{2\pi N}$$

The total width of the distribution is $2N+1$ (from a)

$$\frac{\sqrt{2\pi N}}{2N+1} \rightarrow 0 \text{ when } N \rightarrow \infty$$

When $N = 10^{23}$, this ratio is $\sim 10^{-11}$

Q.30

8) a) $\frac{S}{k} = \ln \Omega = \ln \left(\frac{2^{4N}}{\sqrt{8\pi N}} \right) = 4N \ln 2 - \frac{1}{2} \ln (8\pi N) = 2.7726 \times 10^{23} - 28.1$
 $(N=10^{23})$

b) For most likely macrostate

$$\frac{S}{k} = \ln \left(\frac{2^{4N}}{4\pi N} \right) = 4N \ln 2 - \ln (4\pi N) = 2.7726 \times 10^{23} - 55.5$$

c) The difference between a) and b) is negligible.

Time scale is quite irrelevant for large system.

d) No, it only cause a difference of 27 out of 2.77×10^{23} unit of energy.